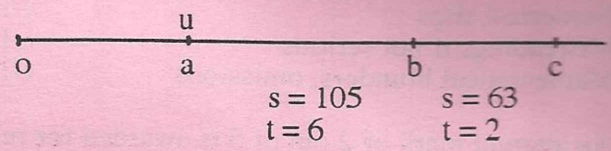
1996

1 (a)



Stage ab $\quad 105 = u(6) + 0.5f(36)$ 5

Stage ac $\quad 168 = u(8) + 0.5f(64)$ 5

$f = 3.5$ 5

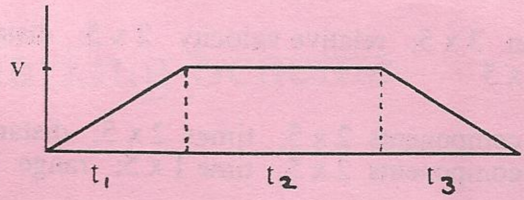
$u = 7$

Stage oa $\quad v^2 = u^2 + 2fs$

$49 = 0 + 2(3.5)s$ 5

$s = 7$ 5 25

(b)



5

$t_1 = \frac{v}{4}$

$t_3 = \frac{v}{4}$ 5

$t_2 = t - t_1 - t_3 = t - \frac{v}{2}$ 5

$d = 0.5t_1 v + t_2 v + 0.5t_3 v$

$= \frac{v^2}{8} + (t - \frac{v}{2})v + \frac{v^2}{8}$ 5

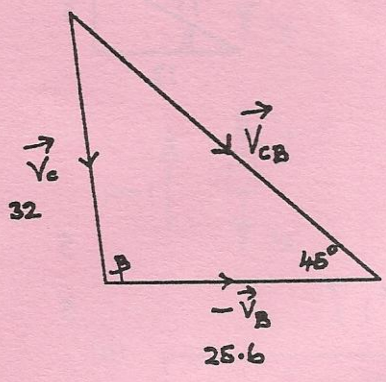
$\Rightarrow \quad v^2 - 4vt + 4d = 0$

$v = 2t \pm 2\sqrt{t^2 - d}$

$\Rightarrow \quad t_2 = t - \frac{v}{2} = \sqrt{t^2 - d}$ 5 25

1996

2 (i) **Relative velocity diagram** 5



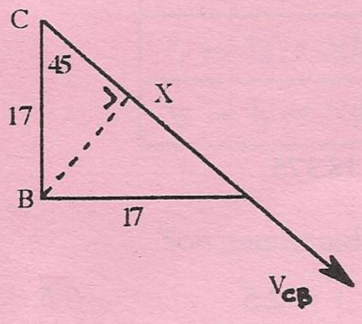
$$\frac{\sin(135-\beta)}{25.6} = \frac{\sin 45}{32}$$ 5

$\Rightarrow \quad 135 - \beta = 34.45°$

$\Rightarrow \quad \beta = 100.55°$ 5 15

(ii) $$\frac{\vec{V}_{CB}}{\sin\beta} = \frac{32}{\sin 45}$$ 5

$\Rightarrow \quad \vec{V}_{CB} = 44.49$ km/h 5 10



(iii) shortest distance $= |BX|$

$= 17 \sin 45$ 5

$= 12.02$ km 5 10

(iv) time $= \dfrac{2\,|CX|}{V_{CB}}$ 5

$= \dfrac{2\,(17\cos 45)}{44.49}$ 5

$= 0.54$ hours 5 15

(i) $\vec{V}_{CB} = x\vec{i} - x\vec{j}$

$\vec{V}_{CB} = \vec{V}_C - \vec{V}_B$ 5

$x\vec{i} - x\vec{j} = \vec{V}_C + 25.6\vec{i}$

$\vec{V}_C = (x - 25.6)\vec{i} - x\vec{j}$

$\Rightarrow \quad 32^2 = (x-25.6)^2 + x^2$

$\Rightarrow \quad x = 31.46$ 5

$\Rightarrow \quad \vec{V}_C = (31.46 - 25.6)\vec{i} - 31.46\vec{j}$

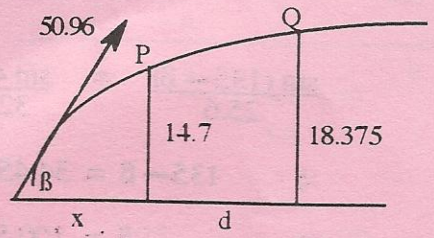
$\Rightarrow$ direction = 79.45° South of East 5 15

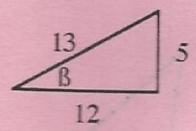
(ii) $\vec{V}_{CB} = 31.46\vec{i} - 31.46\vec{j}$ 5

$\Rightarrow \quad |\vec{V}_{CB}| = 44.49$ km/h 5 10

1996

3 (a)





$\vec{r} = 50.96(\cos\beta)t\,\vec{i}$ 5

$\quad + \{50.96(\sin\beta)t - 0.5gt^2\}\,\vec{j}$ 5

$= 47.04\,t\,\vec{i} + \{19.6\,t - 4.9\,t^2\}\,\vec{j}$

At P $\quad r_{\vec{j}} = 14.7 \Rightarrow 19.6\,t - 4.9\,t^2 = 14.7$

$t^2 - 4t + 3 = 0$

$\Rightarrow \quad t = 1$ 5

At Q $\quad r_{\vec{j}} = 18.375 \Rightarrow 19.6\,t - 4.9\,t^2 = 18.375$

$t^2 - 4t + 3.75 = 0$

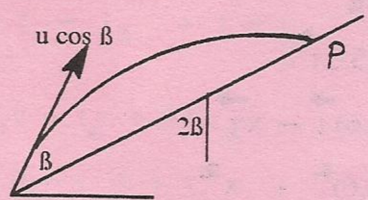
$\Rightarrow \quad t = 1.5$ or $t = 2.5$ 5

When $t = 1$, $\quad x = 47.04(1) \Rightarrow x = 47.04$

When $t = 1.5$, $\quad x + d = 47.04(1.5) \Rightarrow d = 23.52$ 5

When $t = 2.5$, $\quad x + d = 47.04(2.5) \Rightarrow d = 70.56$ 5 30

(b)



$\vec{r} = \{u\cos\beta.\cos\beta.t - 0.5g\cos 2\beta.t^2\}\,\vec{i}$ 5

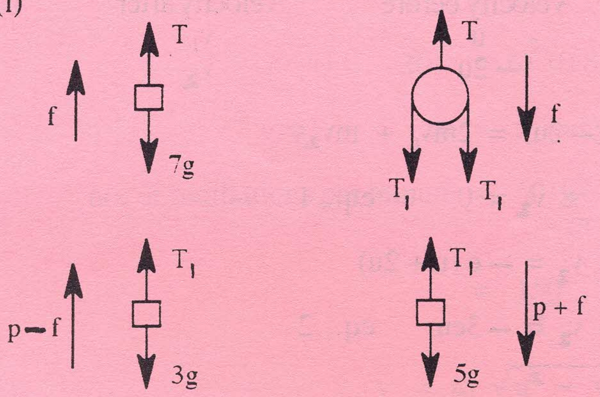
$\quad + \{u\cos\beta.\sin\beta.t - 0.5g\sin 2\beta.t^2\}\,\vec{j}$ 5

At P $\quad r_{\vec{j}} = 0 \Rightarrow t = \dfrac{2u\cos\beta\sin\beta}{g\sin 2\beta} = \dfrac{u}{g}$ 5

Range $= u\cos^2\beta.\dfrac{u}{g} - \dfrac{g}{2}\cos 2\beta.\dfrac{u^2}{g^2} = \dfrac{u^2}{2g}$ 5 20

1996

4 (i)



T, $7g$ and T, T_1, T_1 5

T_1, $3g$ and T_1, $5g$ 5 10

(ii)

$T - 7g = 7f$
$2T_1 - T = 0$
$T_1 - 3g = 3(p - f)$
$5g - T_1 = 5(p + f)$

$T - 7g = 7f$ and $2T_1 - T = 0$ $\Rightarrow$ $2T_1 - 7g = 7f$ eq...1 5

5

$\Rightarrow$ $5T_1 - 15g = 15p - 15f$ 5

$\Rightarrow$ $\underline{15g - 3T_1 = 15p + 15f}$ 5

$8T_1 - 30g = -30f$ eq...2

Solve equations 1 and 2 $\Rightarrow$ $\boxed{f = \frac{g}{29} \text{ or } 0.34}$ 5

$\Rightarrow$ $T_1 = \frac{105g}{29}$ or 35.5

$\Rightarrow$ $T = \frac{210g}{29}$ or 71.0

$\Rightarrow$ $\boxed{p = \frac{7g}{29} \text{ or } 2.4}$ 5 30

(iii)

$T - 7g = 7f$ and $2T_1 - T = 0$ $\Rightarrow$ $2T_1 - 7g = 7f$

$T_1 - mg = m(p - f) = 0$ when $\boxed{p = f}$ 5

$5g - T_1 = 5(p + f)$ $\Rightarrow$ $10g - 2T_1 = 20f$

$\Rightarrow$ $f = \frac{g}{9}$

$T_1 = \frac{35g}{9} = mg$ $\Rightarrow$ $m = \frac{35}{9}$ or 3.9 kg 5 10

1996

5 (a)

mass	velocity before	velocity after
2m	u	v_1
m	$-2u$	v_2

PCM $2m(u) + m(-2u) = 2mv_1 + mv_2$ 5

$2v_1 + v_2 = 0$ eq...1

NEL $v_1 - v_2 = -e(u + 2u)$ 5

$v_1 - v_2 = -3eu$ eq...2

Solve equations 1 and 2

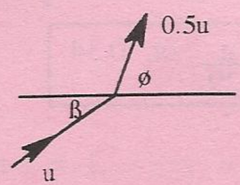
$v_1 = -eu$ and $v_2 = 2eu$ 5

$E_1 = 0.5(2m)u^2 + 0.5(m)(4u^2) = 3mu^2$ 5

$E_2 = 0.5(2m)e^2u^2 + 0.5(m)4e^2u^2 = 3me^2u^2$

$\Rightarrow \frac{E_2}{E_1} = \frac{3me^2u^2}{3mu^2} \Rightarrow e = \sqrt{\frac{E_2}{E_1}}$ 5 25

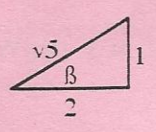
(b)



velocity in vertical direction does not change, therefore

$u \sin ß = 0.5u \sin ø$

$\sin ø = 2\sin ß = \frac{2}{\sqrt{5}}$ 5



$\Rightarrow \cos ø = \frac{1}{\sqrt{5}}$

PCM $mu\cos ß + m(0) = m(0.5u\cos ø) + mv_2$ 5

NEL $0.5u\cos ø - v_2 = -e\{u\cos ß - 0\}$ 5

$0.5u\cos ø + v_2 = u\cos ß$

$0.5u\cos ø - v_2 = -eu\cos ß$

$u\cos ø = (1 - e)u\cos ß$ 5

$\frac{1}{\sqrt{5}} = (1-e)\frac{2}{\sqrt{5}}$

$\Rightarrow e = 0.5$ 5 25

1996

6(a) (i)

$$f = \omega^2 x$$

$$20 = \omega^2 (0.8)$$

$$\Rightarrow \quad \omega = 5 \text{ rad/s}$$ 5

no. of oscillations per minute $= 60 \dfrac{\omega}{2\pi}$

$= \dfrac{150}{\pi}$ or 47.7 5 10

(ii)

$$v = \omega \sqrt{a^2 - x^2}$$

$$2 = 5\sqrt{a^2 - 0.64}$$

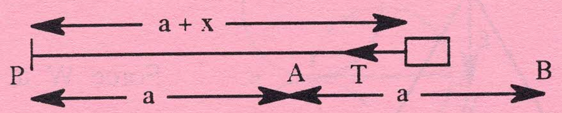
$\Rightarrow \quad a = \sqrt{0.8}$ or 0.89 m 5 5

(iii) max. $f = \omega^2 a = 25\sqrt{0.8}$ 5

Force $= m f$

$= 250\sqrt{0.8}$ or 223.6 N 5 10

(b) (i)



Force in dirn of x inc $= -T$

$= -kx$ 5

acceleration $= -\dfrac{k}{m}x$

Therefore S.H.M. 5 10

about $x = 0$ with $\omega = \sqrt{\dfrac{k}{m}}$

(ii) time to travel from B to A $= \dfrac{\text{Period}}{4}$

$= \dfrac{2\pi}{4\omega} = \dfrac{\pi}{2}\sqrt{\dfrac{m}{k}}$ 5

velocity at A $= \omega a$ 5

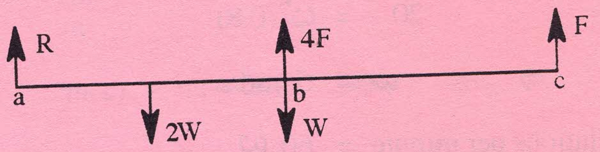
time to travel from A to P $= \dfrac{\text{distance}}{\text{velocity}}$

$= \dfrac{a}{\omega a} = \sqrt{\dfrac{m}{k}}$

$\Rightarrow$ total time $= \left(\dfrac{\pi}{2} + 1\right)\sqrt{\dfrac{m}{k}}$ 5 15

1996

7 (a)



Moments about a

$4F(1) + F(2) =$ 5

$2W(0.5) + W(1)$ 5

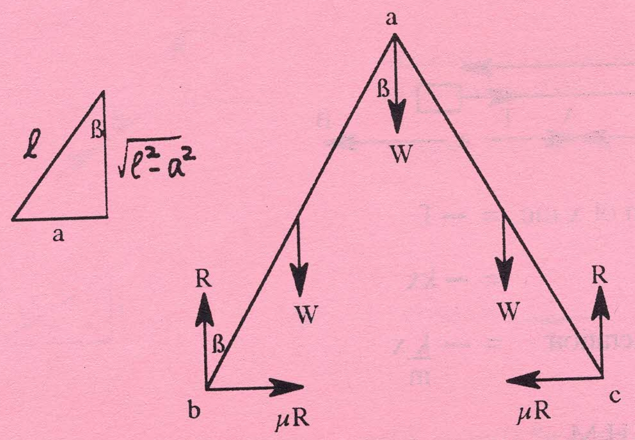
$\Rightarrow \quad F = \dfrac{W}{3}$ 5

Resolve vertically

$R + 5F = 3W$

$\Rightarrow \quad R = \dfrac{4W}{3}$ 5 20

(b)



Force W at a 5

Other forces 5

Resolve vertically: $2R = 3W$ 5

$R = \dfrac{3W}{2}$

Moments about a for rod ab

$\mu R.\ell\cos\beta + W.\tfrac{1}{2}\ell\sin\beta =$ 5

$R.\ell\sin\beta$ 5

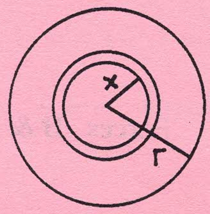
$\Rightarrow \quad \mu R + \tfrac{1}{2}W\tan\beta = R\tan\beta$

$\mu R = \dfrac{2}{3}R\tan\beta$

$\mu = \dfrac{2a}{3\sqrt{\ell^2 - a^2}}$ 5 30

1996

8 (a)



Let m = mass per unit area

mass of element = m ($2\pi x.dx$)

Moment of Inertia of element = $(2\pi m\, x.dx)\, x^2$ 5

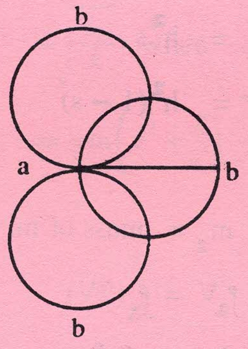
$$I = 2\pi m \int_0^r x^3\, dx$$ 5

$$= 2\pi m \left[\frac{x^4}{4}\right]_0^r$$ 5

$$= \pi m \frac{r^4}{2}$$

$$= \frac{M r^2}{2} \quad \text{where } M = m\pi r^2$$ 5 20

(b)



(i) $I = 0.5\, m r^2 + m r^2 = \frac{3}{2} m r^2$ 5

Gain in K.E. = Loss in P.E.

$$0.5\, I \omega^2 - 0.5\, I \frac{p^2}{4r^2} = m g r$$ 5

$$\frac{3}{4} m r^2 \omega^2 - \frac{3}{4} m r^2 \frac{p^2}{4r^2} = m g r$$ 5

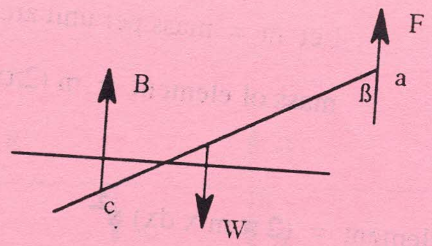
$$\omega = \sqrt{\frac{16 g r + 3 p}{12 r}}$$ 5 20

(ii) Loss in K.E. = Gain in P.E.

$$\frac{1}{2}\,\frac{3mr^2}{2}\,\frac{p^2}{4r^2} = mgr$$ 5

$$p = \sqrt{\frac{16gr}{3}} \quad \text{or} \quad 4\sqrt{\frac{gr}{3}}$$ 5 10

9 (a)



Forces B & W 5

$$|ac| = q + \frac{(h-q)}{2} = \frac{h+q}{2}$$ 5

$$B = \frac{\frac{(h-q)W(1)}{h}}{s} = \frac{(h-q)W}{hs}$$ 5

Moments about a

$$B\,|ac|\sin\beta = W(0.5h)\sin\beta$$

$$\frac{(h-q)W}{hs}\frac{(h+q)}{2} = \frac{Wh}{2}$$ 5

$$\Rightarrow \quad h^2 - q^2 = h^2 s$$

$$q^2 = h^2(1-s)$$ 5 25

(b) Equal volumes: $m_1 + m_2 =$ mass of mixture

$$\rho_1 V + \rho_2 V = \rho_3(2V)$$ 5

$$\rho_1 + \rho_2 = 2\rho_3$$ 5

$$s_1 + s_2 = 5$$

Equal weights: $V_1 + V_2 =$ Volume of mixture

$$\frac{m}{\rho_1} + \frac{m}{\rho_2} = \frac{2m}{\rho_4}$$ 5

$$\frac{1}{s_1} + \frac{1}{s_2} = \frac{2}{2.4}$$ 5

$$s_1 + s_2 = \frac{2(s_1 s_2)}{2.4}$$

$$s_1 s_2 = 6$$

$$\Rightarrow \quad s_1 = 2 \quad \text{and} \quad s_2 = 3$$ 5 25

1996

10 (a)

$$\int \frac{dy}{y} = 4\int \cos x\,dx \qquad 5$$

$$\ln y = 4\sin x + c \qquad 5$$

$$y = e^2 \text{ when } x = \frac{\pi}{6} \Rightarrow 2 = 4(\tfrac{1}{2}) + c$$

$$\Rightarrow c = 0 \qquad 5$$

$$\therefore \ln y = 4\sin x$$

$$\Rightarrow y = e^{4\sin x} \qquad 5 \qquad 20$$

(b) (i) particle moving upwards

$$mv\frac{dv}{dx} = -mkv^2 - mg$$

$$\int\left(\frac{v\,dv}{v^2 + \frac{g}{k}}\right) = -k\int dx$$

$$\tfrac{1}{2}\ln(v^2 + \tfrac{g}{k}) = -kx + C \qquad 5$$

$$v = \sqrt{\frac{2g}{k}} \text{ when } x = 0 \Rightarrow \frac{1}{2}\ln\left(\frac{3g}{k}\right) = 0 + C$$

$$\Rightarrow \tfrac{1}{2}\ln\left(v^2 + \frac{g}{k}\right) = \tfrac{1}{2}\ln\left(\frac{3g}{k}\right) - kx \qquad 5$$

Find x when v = 0

$$\tfrac{1}{2}\ln\left(\frac{g}{k}\right) = \tfrac{1}{2}\ln\left(\frac{3g}{k}\right) - kx$$

$$kx = \tfrac{1}{2}\ln 3$$

$$x = \frac{\ln 3}{2k} \qquad 5 \qquad 15$$

10 (b) (ii) particle moving downwards 1996

$$mv\frac{dv}{dx} = mg - mkv^2$$

$$\int\left(\frac{v\,dv}{\frac{g}{k} - v^2}\right) = k\int dx$$

$$-\tfrac{1}{2}\ln\left(\frac{g}{k} - v^2\right) = kx + C$$ 5

$v = 0$ when $x = 0 \Rightarrow -\frac{1}{2}\ln\left(\frac{g}{k}\right) = 0 + C$

$$\Rightarrow \tfrac{1}{2}\ln\left(\frac{g}{k}\right) - \tfrac{1}{2}\ln\left(\frac{g}{k} - v^2\right) = kx$$ 5

Find v when $x = \frac{\ln 3}{2k}$

$$\tfrac{1}{2}\ln\left(\frac{g}{k}\right) - \tfrac{1}{2}\ln\left(\frac{g}{k} - v^2\right) = \frac{\ln 3}{2}$$

$$\ln\left(\frac{\frac{g}{k}}{\frac{g}{k} - v^2}\right) = \ln 3$$

$$v = \sqrt{\frac{2g}{3k}}$$ 5 15